An automatic stabilization system for quadrotors with applications to vertical take-off and landing

Ilan Zohar, Amit Ailon, and Hugo Guterman

Abstract—In the last decade many studies have been conducted in the technological framework of Unmanned Aerial Vehicles (UAVs). There is great interest in developing autonomous UAVs, in particular, unmanned vertical take-off and landing (UVTOL) aircraft, because of their ability to conduct search and rescue missions in remote dangerous areas, without endangering human operators. The highly complicated flight control system of UVTOL aircraft presents a serious challenge for scientists. In this research we developed a control algorithm for quadrotor attitude and altitude stabilization. The system, which includes advanced hardware for real-time data processing, is reliable and robust and thus ensures safe flight and take-off and landing. In the current study we implement the control scheme in an analytical model of a quadrotor platform and demonstrate vertical take-off and landing and hovering. Later, the realization of the proposed controller will be based on microsensors and microelectronic circuits and the implementation of the control laws will be done using advanced sensing systems including IMU and pressure devices.

Keywords—Quadrotor, Lyapunov stability, attitude and altitude dynamics and control.

I. INTRODUCTION

Flying objects have always been a fascinating topic for scientists of various disciplines, encouraging all kinds of research and development. The field of applications is broad in both military and civilian fields and markets. The ability to execute unmanned missions (patrol or surveillance) and track moving targets, while supporting the communication and control systems, makes the technological framework of flying objects a real scientific challenge. Unmanned Vertical Take-Off and Landing (UVTOL) design and control require the development and implementation of complex algorithms to control the mechanical nonlinear system.

Precise knowledge of the quadrotor position and orientation and kinematic data is needed. This information can be obtained from inertial navigation systems (INS), global positioning systems (GPS), and/or other devices such as sonar and vision sensors. Vision sensors are primarily used for estimating the relative positions of some target, such as a landing site or a moving ground vehicle. Unfortunately, a vision system is not as fast as a gyro and requires a great deal of processing resources. Furthermore, a vision-based sensor is not as reliable as other sensors due to its sensitivity to changes in lighting conditions. Typically, multiple sensors are used to overcome limitations of individual sensors, thereby increasing reliability and decreasing errors. A popular modern sensing technology, consisting of a new generation of integrated micro inertial measurement unit (IMU) composed of micro-electromechanical systems (MEMS) technology [1], inertial and magneto-resistive sensors. MEMS technology is less accurate than conventional sensor technology because of noise and drift [2], [3].

The quadrotor is an under-actuated dynamic vehicle with four input forces. Unlike regular helicopters that have variable pitch angle rotors, a quadrotor helicopter has four fixed-pitch angle rotors. The quadrotor motion is generated by varying the rotor speeds of the rotors, thereby changing the aircraft lift force and attitude. The quadrotor tilts toward the direction of the slow spinning rotor, which enables acceleration along that direction. Therefore, control of the tilt angles and the motion of the helicopter are closely related. The spinning directions of the rotors are set to balance the moments and eliminate the need for a tail rotor. This principle is also used to produce the desired yaw motions. A good controller should properly arrange the speed of each rotor so that the desired states change. Integrating sensors, actuators and computing units into a lightweight vertically flying system is a major technological challenge. The state-of-the-art in the quadrotor control technology has drastically changed during the last few years [4], and the number of projects tackling this problem has considerably increased. Most of these projects are based on commercially available devices like the draganflyer [5], modified afterwards to have more sensory and communication capabilities. The work of [6] has tackled simultaneous consideration of design and control for a quadrotor.

This study performs a comprehensive analysis of the model system in terms of mathematical properties and characterization. The relevant control laws will
be developed analytically, and the proposed control schemes will be studied and evaluated numerically. These control laws will be realized in the future in UVTOL using an existing lab prototype. Implementation of the control laws will be done using IMU, and ultrasonic and barometer devices. The sensors should provide accurate information on the UVTOL height, ultrasonic and barometer devices. The sensors should provide accurate information on the UVTOL height, angular condition, and its velocity and space orientation. Controllers will first be developed for executing safe vertical take-off and landing. The present analysis deals mainly with the issue of rigid body equations of motion and control. Then, once we have established some relevant controllers for the model of a rigid body that moves in a 3D space, we shall consider and apply the obtained results for the associated control problem in the underlying quadrotor model.

II. MODELING

A. Dynamic Modeling

The evolution of the system configuration can be described by the center of mass motion and the motion about the center of mass. Let \( F \) be the resultant vector of the external forces acting on the body, and \( M \) the moment of these forces with respect to the mass center; the motion is governed by the equation

\[
\begin{align*}
\frac{dv}{dt} &= F \\
\frac{dL}{dt} &= M
\end{align*}
\]

(1)

where \( v \) is the velocity of the mass center relative to the inertial frame \( \{x, y, z\} \), \( L \) is the angular momentum relative to the same frame, and \( m \) is the total mass of the rigid body.

Let the coordinate system of the principal axes \([7]\) be attached to the rigid body in Fig. 1. Assume that the origin of the rigid body frame is located at the rigid body mass center. Recall that the inertial coordinate system is assigned by \( \{x, y, z\}_i \) and the rigid body coordinate system by \( \{\alpha, \beta, \gamma\}_b \). Since the unit vectors \( i_b, j_b, k_b \) are time varying (and thus their time derivatives do not vanish) we have

\[
\begin{align*}
\frac{dv_b}{dt} &= \dot{v}_b + \omega_b \times v_b \\
\frac{dL_b}{dt} &= \dot{L}_b + \omega_b \times L_b.
\end{align*}
\]

(2)

Hence, (1) in terms of the body frame is given by

\[
\begin{align*}
m\dot{v}_b + \omega_b \times mv_b &= F_b \\
J\dot{\omega}_b + \omega_b \times J\omega_b &= M_b
\end{align*}
\]

(3)

where (recall that we consider the case of principle axes)

\[
J = \begin{bmatrix}
J_\alpha & 0 & 0 \\
0 & J_\beta & 0 \\
0 & 0 & J_\gamma
\end{bmatrix}.
\]

(4)

The above equations describe how the forces and moments affect the translational and rotational velocity of the rigid body. We should, however, also develop the kinematic equations, which allow us to relate rotational velocities to attitude. To start with we wish to have the coordinates of the motion path relative to the inertial frame. We first apply a sequence of rotations using the Euler angles \( \psi, \theta, \phi \). We made a three sequence rotation: 1) A rotation \( \psi \) about the axis \( oz \) carrying the axes \( \{x, y, z\} \) to \( \{x_1, y_1, z_1\} \). 2) A rotation \( \theta \) about \( oy_1 \) carrying the system \( \{x_1, y_1, z_1\} \) to \( \{x_2, y_2, z_2\} \). 3) A rotation about the \( ox_2 \) carrying the system \( \{x_2, y_2, z_2\} \) to the body frame \( \{\alpha, \beta, \gamma\}_b \). The aircraft velocity in terms of the inertial coordinate system is given by

\[
v_i = \dot{x}i + \dot{y}j + \dot{z}k_i
\]

(5)

and by using Euler angles \( \{\psi, \theta, \phi\} \) and the associated sequence of rotations (for details see, e.g., [8]) we have

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix}_i = R_b^i v_b,
\]

(6)

where the subscripts \( i \) and \( b \) denote, respectively, the inertial and rigid body frames, and

\[
R_b^i = \begin{bmatrix}
\cos\phi \cos\psi & \cos\phi \sin\psi & -\sin\phi \\
-cos\theta \sin\psi + s \cos\phi \sin\psi & c \cos\theta \cos\psi + s \sin\phi \sin\psi & -c \sin\theta \cos\psi + s \sin\phi \cos\psi \\
-cos\theta \sin\psi - s \cos\phi \sin\psi & c \cos\theta \cos\psi - s \sin\phi \sin\psi & c \sin\theta \cos\psi - s \sin\phi \cos\psi
\end{bmatrix}
\]

(7)

Remark. Note that the sequence of carrying out the Euler rotations is not always the same in the relevant references and, to date, no standard sequence has been agreed upon. Here we adhere to the sequence of rotations that are mentioned above [8, Section 4.5], which is different than the ones in [7] and [9].
Next we wish to express the orientation of the quadrotor as a function of the angular velocity components \( \{p, q, r\} \). In terms of the rigid body coordinate system we have

\[
\omega_b = p_i b + q_j b + r_k b. \tag{8}
\]

The relation between \( \omega_b \) and the rate of change of the Euler angles is now considered. In the first step in the sequence of rotations from the inertial frame to the body frame (see above), the rate of rotation is

\[
L_{bi}^{-1} \dot{L}_{ib} = \begin{bmatrix}
1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi \sec \theta & \cos \phi \sec \theta
\end{bmatrix} \tag{9}
\]

since \( \det L_{bi} = \cos \theta \) the relationship between the angular velocity and the Euler angle rates can be inverted provided that \( \theta \neq \pi/2 \). Assuming this is the case,

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} = L_{ib} \begin{bmatrix}
p \\
q \\
r
\end{bmatrix}. \tag{10}
\]

Note that the matrix \( L_{bi} \) is not a rotation matrix and \( L_{bi} \neq L_{ib}^{-1} \).

**B. Equation of motion**

Defining \( \xi = [\phi, \theta, \psi]^T \), the complete model of the equations of motion of a rigid body is given using (3), (4), and (10) by

\[
\begin{align*}
\dot{v}_b &= -\omega_b \times v_b + F_b/m \\
\dot{\xi} &= L_{ib}(\xi)\omega_b \\
\dot{\omega}_b &= -J^{-1}\omega_b \times J\omega_b + J^{-1}M_b. \tag{11}
\end{align*}
\]

At this stage the presented analysis refers mainly to the rigid body equations of motion. Once we have established controllers for achieving basic control objectives such as attitude control and asymptotic stability and tracking in the rigid body model, we will extend and apply the results for the associated control problems in the quadrotor system.

Since the considered rigid body is a flying object, the force \( F_b \) and the moment \( M_b \) are the components of the total aerodynamic forces (including propulsive forces) and the components of the weight and moments. We represent the force and moments as:

\[
F_b = [f_a + w_\alpha, f_\beta + w_\beta, f_\gamma + w_\gamma]^T
\]

\[
M_b = [M_\alpha, M_\beta, M_\gamma]^T \tag{12}
\]

where \( W_b = [w_\alpha, w_\beta, w_\gamma]^T \) is the force due to the gravitational field in terms of the rigid body frame. The force of gravity is given in the body frame using (7) by

\[
W_b = R_b^T W_i = mg \begin{bmatrix}
-\sin \theta \\
\cos \theta \sin \phi \\
\cos \theta \cos \phi
\end{bmatrix}. \tag{13}
\]

**III. ATTITUDE CONTROL FOR RIGID BODY**

It is important to note that the second and third equations in (11) are independent of the first one. From matrix algebra the skew symmetric matrix

\[
S(a) = \begin{bmatrix}
0 & -a_\gamma & a_\beta \\
a_\gamma & 0 & -a_\alpha \\
-a_\beta & a_\alpha & 0
\end{bmatrix} \tag{14}
\]

satisfies \( S(a) w = a \times w = -w \times a \) for any pair of vectors \( \{a, w\} \). Hence, we may rewrite the last two equations of (11) as

\[
\begin{align*}
\dot{\xi} &= L_{ib}(\xi)\omega_b \\
\dot{\omega}_b &= J^{-1} S(J\omega_b)\omega_b + J^{-1}M_b. \tag{15}
\end{align*}
\]

The rigid body attitude determines its orientation with respect to an inertial frame. The attitude stabilization problem is associated with the design of a controller that ensures \( \lim_{t \to \infty} \xi(t) \to \xi_d \) (a constant vector) and \( \lim_{t \to \infty} \omega_b(t) = 0 \) (the zero vector); namely, the design ensures stabilization of the rigid body attitude with respect to a desired reference point \( [\xi^T, \omega_b^T] = [\xi_d^T, 0]^T \in \mathbb{R}^6 \). Concerning attitude control, two problems should be considered: (i) stabilization of an attitude set-point (a fixed reference attitude) and (ii) tracking a time-varying reference attitude. The set-point attitude regulation is an extremely important issue in various aircraft control problems. For example, in air photo missions the camera orientation depends on the aircraft attitude. Therefore, any change in the attitude during imaging might cause camera orientation errors and angle deviation, and hence distortion of pictures.

For the attitude tracking problem assuming the desired reference attitude trajectory is \( \xi^* (t) \), the control objective is quantified by the attitude tracking error \( e(t) \in \mathbb{R}^3 \), which is defined as \( e(t) = \xi(t) - \xi^*(t) \). The goal is accomplished if the applied controller ensures

\[
\lim_{t \to \infty} e(t) = 0. \tag{16}
\]

Attitude tracking is important for control tasks such as air photos, trajectory tracking, and formation control.

A simple state controller for stabilizing a set-point (see (i) above) will be demonstrated. (Note that we are going to solve the problem by taking the full nonlinear model into consideration, and thus system linearization will not be needed). The proposed controller, which in a way resembles a proportional derivative controller, follows from standard tools in the nonlinear system theory [9, Section 8.2], [10], of robotic-like systems.

We demonstrate an approach for obtaining a simple state controller for attitude regulation. Without loss of generality, we may assume that \( \xi_d = 0 \). If this is not
the case and \( \xi_d \neq 0 \), one may apply the change of variables \( \zeta = \xi - \xi_d \) and replace the first equation in (15) by \( \dot{\zeta} = L_{ib}(\zeta + \xi_d)\omega_b \) and continue as follows with \( \zeta \) replacing \( \xi \). From [11] we have the following statement and proof, that will play a central role in the design of the quadrator’s attitude/altitude controller.

**Lemma 1.** Consider (15) and let the applied torque be

\[
M_b = - \left( L_{ib}^T (\xi) K \xi + B \omega_b \right) \tag{17}
\]

where \( K = K^T \), \( B = B^T \) are any selected constant positive definite matrices. Then the trivial solution of (15) is asymptotically stable.

**Proof.** Consider a Lyapunov candidate function

\[
V(\xi, \omega_b) = \frac{1}{2} \left[ \xi^T K \xi + \omega_b^T J \omega_b \right] \tag{18}
\]

where \( K = K^T > 0 \). Observing (17) and plugging \( M_b \) to (15), the derivative of \( V \) along the trajectories of (15), namely, \( \dot{V} = \xi^T K \xi + \omega_b^T J \omega_b \) satisfies

\[
\dot{V} = \xi^T K L_{ib}(\xi) \omega_b + \omega_b^T L_{ib}^T (\xi) K \xi - \omega_b^T B \omega_b = -\omega_b^T B \omega_b \leq 0. \tag{19}
\]

The second equality follows from the facts that \( S(J_{ib}) \) is skew symmetric (and hence for each \( \omega_b \in \mathbb{R}^3 \) one has \( \omega_b^T S(J_{ib}) \omega_b = 0 \)) and \( \xi^T K L_{ib}(\xi) \omega_b = \omega_b^T L_{ib}^T (\xi) K \xi \). Invoking the LaSalle’s invariance principle [12] the lemma follows. ◆

**IV. STABILIZING CONTROLLERS FOR THE QUADROTOR**

**A. preliminaries**

In this case we consider the rigid body as a flying object, then the force \( F_b \) and the moment \( M_b \) are defined by two parameters: the configuration of the motor and the trust developed by each motor/propeller. In Fig. 1 the circular arrows indicate the direction of rotation of each propeller. As we can see from (12), we need to define \( f_b = [f_x, f_y, f_z]^T \) and \( M_b = [M_x, M_y, M_z]^T \). It is well known that in quadrator the motors are positioned with the rotor at \(-k_b\) with respect to the body system, and the motor velocity is the only controlled parameter.

The propeller thrust is produced by pushing air in a direction perpendicular to its plane of rotation [13]. The air flow generates thrust to push the aircraft in reaction to the air drag on the blades. The thrust/lift force is

\[
f_i = -b \omega_i^2 k_b \tag{20}
\]

where \( b \) is a positive constant that depends on the fluid density, flow velocity, lift coefficient, and propeller parameter.

The drag force, which is parallel to the blade direction, is

\[
\tau_i = -k \omega_i^2 k_b \tag{21}
\]

where \( k \) is a positive constant that depends on the fluid density, flow velocity, drag coefficient, and the propeller parameters.

As the blades rotate around axes that are at distance \( l \) from the center of the aircraft, the drag force produces a torque around the center of the aircraft equal to \( l \cdot f_i \); with this knowledge we can control the torque by changing the speed of the pair motors. As the two pairs of motors rotate in opposite directions, these torques will balance to zero through the airframe when the motors are turning at the same velocity. When commanded to yaw, the controller decreases the velocity of one pair to create a torque imbalance that causes the quadrator to turn around the \( k_b \) axis. Let us redefine the set of equations (12) using (20) and (21) to get

\[
F_b = \left[ 0 + w_x, 0 + w_y, u + w_z \right]^T
\]

\[
M_b = \begin{bmatrix} f_1 - f_3 \xi & \left( f_1 - f_3 \right) \xi \\ \left( f_1 - f_3 \right) \xi & \tau_2 + \tau_3 - \tau_1 \end{bmatrix} \tag{22}
\]

where \( u = \sum f_i = -b \omega_i^2 \), the thrust of all motors. \( \xi \)

From (22) we can see that using (17) as the control law we achieve attitude control.

**B. Stabilizing controllers for attitude and altitude**

This sub-section is devoted to developing an altitude/attitude controller for the underlying system. First, it can be shown that the system acceleration in the \( x, y, \) and \( z \) directions is given by

\[
\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \frac{1}{m} \begin{bmatrix} cv \theta \sin \phi + s \phi \cos \psi \\ s \phi \theta \sin \phi - c \phi \cos \psi \\ c \theta \phi \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}. \tag{23}
\]

The current control objective is to stabilize the system with respect to a desired altitude while maintaining the attitude in almost zero position; that is, \( z(t) \rightarrow z_d \) where \( z_d \) is the desired altitude and the Euler angles vector satisfies \( \xi \rightarrow 0 \).

We synthetize the controller by applying the feedback linearization technique. To this end let

\[
u = m \frac{h - ng \cos \theta \cos \phi}{c \theta \phi} \tag{24}
\]

where

\[
h = -k_1 (z - z_d) - k_2 \dot{z}. \tag{25}
\]

Implementing (24)-(25) in (23) the last equation reduces to

\[
\ddot{z} = -k_3 (z - z_d) - k_4 \dot{z}. \tag{26}
\]

It is straightforward to prove that provided \( \cos \theta(t) \cos \phi(t) \neq 0 \) for all \( t \geq 0 \), we have \( z \rightarrow z_d \), \( \dot{z} \rightarrow 0 \) exponentially for all \( k_1, k_2 > 0 \).

Hence, it is necessary to state a condition (in the initial state) that guarantees \( \cos \theta(t) \cos \phi(t) \neq 0 \) during the process and ensures that (24) is well defined.
For this we recall Lemma 1 and rewrite the Lyapunov function (18) as

\[ V(\gamma) = \frac{1}{2} \gamma^T H \gamma \]  

(27)

where \( \gamma \in [\xi^T, \omega_b^T]^T \in \mathbb{R}^6 \) and \( H \) is a real block diagonal matrix

\[ H = \begin{bmatrix} K & 0 \\ 0 & J \end{bmatrix} \in \mathbb{R}^{6 \times 6}. \]  

(28)

Note that by definition \( H = H^T > 0 \). Let the minimum and maximum eigenvalues of \( H \) be defined by \( \lambda_{\min} \) and \( \lambda_{\max} \), respectively. Clearly \( 0 < \lambda_{\min} \leq \lambda_{\max} \).

Fix \( 0 < \rho < \pi/2 \) and define \( r \)

\[ r \doteq \sqrt{\frac{\lambda_{\min}}{\lambda_{\max}}} (\pi/2 - \rho). \]  

(29)

We claim that

\[ \|\gamma(0)\| \leq r \Rightarrow \|\xi(t)\| \leq \|\gamma(t)\| \leq (\pi/2 - \rho); \forall t \geq 0. \]  

(30)

To this end we recall that \( \dot{V}(\gamma(t)) \leq 0 \) and hence \( V(\gamma(t)) \) is non-increasing function and one may write

\[ V(\gamma(t)) \leq V(\gamma(0)); \forall t \geq 0. \]  

(31)

In addition we know that for each \( \gamma \)

\[ \lambda_{\min} \|\gamma\|^2 \leq V(\gamma) \leq \lambda_{\max} \|\gamma\|^2. \]  

(32)

Equations (31) and (32) state that

\[ \lambda_{\min} \|\gamma(t)\|^2 \leq V(\gamma(t)) \leq \lambda_{\max} \|\gamma(0)\|^2 \]  

and since \( \|\xi(t)\| \leq \|\gamma(t)\| \), if (30) holds we have from (33) (see also (29)):

\[ \lambda_{\min} \|\xi(t)\|^2 \leq \lambda_{\min} \|\gamma(t)\|^2 \]

\[ \leq \lambda_{\max} \left(\sqrt{\frac{\lambda_{\min}}{\lambda_{\max}}} (\pi/2 - \rho)\right)^2 \]

\[ = \lambda_{\min} (\pi/2 - \rho)^2 \]  

(34)

and the claim follows. Note in particular that (30) assures that \( |\theta(t)|, |\phi(t)| \leq \|\xi(t)\| \leq (\pi/2 - \rho) \) for all \( t \geq 0 \) and hence (24) is well defined provided the initial condition of the system (15) satisfies \( \gamma(0) \leq \sqrt{\lambda_{\min}/\lambda_{\max}} (\pi/2 - \rho) \) for any fixed \( \rho \in (0, \pi/2) \).

V. SIMULATION RESULTS

A. Platform details

Fig. 2 shows a mikrokopter [14] open source quadrotor equipped with sensors and computational devices. For future experiments our system consists of a custom built aluminum frame with carbon fiber landing gear. The navigation and control system is based on a MEMS inertial sensor and a processor board that hosts the closed loop stabilization routines. For a ground station control, a data modem (2.4 GHZ) was installed to communicate between the ground station and the quadrotor. Propulsion is generated by four direct current (DC) brushless model motors located at the ends of each cross-frame arm. Each motor is capable of sustaining a continuous 20 amps. The propellers are mounted directly to the motor shaft, thus eliminating the complexity and weight associated with gear boxes. Power for each motor is provided by a lithium polymer (Li-Po) battery pack rated for 80 amps continuous current and featuring an 4 amp-hour capacity. A carbon fiber structure attached to the bottom of the cross-frame serves as both landing gear and as a protective enclosure for the camera. The camera is mounted inverted to the bottom of the cross-frame where it has a 360° field of view obstructed only by four vertical carbon-fiber rods that extend down to the landing gear. A 900 Mhz down link was installed in order to get the stream of the video in real-time in the ground station. Complementary filters are used to filter the MEMS data and a digital LPF use to filter the pressure and the ultra sonic sensor. The control system runs at a sample rate of 488 Hz.

B. Simulation results

Example 1: In this case, the quadrotor was tasked to stabilize its attitude while maintaining a constant altitude. The considered initial condition is \( \xi(0) = \)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>0.9[kg]</td>
<td>( l )</td>
<td>1[m]</td>
</tr>
<tr>
<td>( I_x )</td>
<td>0.0625[kg \cdot m^2]</td>
<td>( I_y )</td>
<td>0.0625[kg \cdot m^2]</td>
</tr>
<tr>
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<td>( T_z )</td>
<td>1[mSec]</td>
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<tr>
<td>( k )</td>
<td>1</td>
<td>( b )</td>
<td>1</td>
</tr>
<tr>
<td>( K )</td>
<td>30</td>
<td>( B )</td>
<td>30</td>
</tr>
<tr>
<td>( k_1 )</td>
<td>4</td>
<td>( k_2 )</td>
<td>4</td>
</tr>
</tbody>
</table>

TABLE I

QUADROTOR SIMULATION PARAMETERS

While various experiments will be conducted in the near future by the aerial robotics research group here, we shall be presenting simulation results that demonstrate the performance of the designed flight controllers. The simulation (using Simulink) results are obtained based on the data of Table I associated with the quadrotor that was described above:
[π/6, π/6, π/6] (angles are in radians) and the prespecified height is \(z_d = -10[m]\) (recall that in a vertical position the positive direction of the \(z\)-axis points towards the ground). Figure 3 and Figure 4 demonstrate set-point tracking and the quadrotor altitude. The results show how the attitude converges to the zero state and the desired height \(z_d\) is preserved.

Example 2: In this three-phase simulation, the quadrotor was tasked to take-off, hover, and finally land. The quadrotor initial attitude is \(\xi(0) = [0, 0, \pi/4]\). The desired height was \(z_d = -10[m]\). The control objective is first to ensure \(\xi(0) \rightarrow 0\) and \(z \rightarrow z_d\), and then \(z \rightarrow 0\) while maintaining \(\xi = 0\). Figure 5 shows the quadrotor in taking-off, hovering, and landing.

![Fig. 3. Time history of the quadrotor attitude (\(\xi\)) while maintaining a constant altitude.](image)

![Fig. 4. Time history of the attitude speed (\(\dot{\xi}(t)\)) while maintaining a constant altitude.](image)

VI. CONCLUSIONS

The main goal of this research is to develop a nonlinear controller for stabilizing the quadrotor attitude and altitude during flight. We started by the development of the nonlinear kinematic/dynamic model of the quadrotor, and then have established a relatively simple controller for the obtained highly nonlinear system.

Simulation results demonstrate the controller potential and performances. Further research will be focusing on developing control schemes for various aerial tasks of the considered system, in particular for trajectory tracking under conditions of model uncertainties.

REFERENCES